

# **Conformal Invariance and Gravitational Coupling in Quantum Field Theory**

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We study the quantum constraints of a conformal invariant action for a scalar field. For this purpose we briefly present a reformulation of the duality principle advanced earlier in the context of generally covariant quantum field theory, and apply it to examine the finite structure of the quantum constraints. This structure is shown to admit a dimensional coupling (a coupling mediated by a dimensional coupling parameter) of states to gravity. Invariance breaking is introduced by defining a preferred configuration of dynamical variables in terms of the large-scale characteristics of the universe. In this configuration a close relationship between the quantum constraints and the Einstein equations is established.

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## **1. INTRODUCTION**

There is an open possibility that the gravitational coupling of matter may have its origin in an actual invariance-breaking effect of some fundamental symmetry of nature. It seems clear that the establishment of such a possibility would unquestionably improve greatly our views about the nature of gravity.

Concerning a theory of this type, there is an important remark indicating the kind of symmetry which may be of significance. In fact, since the ordinary coupling of matter to gravity is a dimensional coupling (mediated, namely, by the gravitational constant), those local transformations which could change the strength of this dimensional coupling are expected to play a key role. As a consequence one may consider the principle of conformal invariance as a fundamental invariance principle. The corresponding transformations are viewed to affect the local standards of length and time in a measurement process by a position-dependent multiplicative factor  $\Omega(x)$  applied to them.

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The standard of mass changes by the inverse factor (the velocity of light and the Planck constant are unaffected). The corresponding effect on the space-time metric can be represented by the law<sup>4</sup>

$$g_{\mu\nu} \rightarrow \Omega^2(x) g_{\mu\nu} \quad (1)$$

The factor  $\Omega(x)$  is assumed to be an arbitrary, positive, and smooth space-time function. Deser (1970) applies the principle of conformal invariance<sup>5</sup> and a cosmological invariance-breaking effect to a classical scalar field, and shows that the resulting form of the gravitational coupling of matter is as suggested by general relativity. In more specific terms, Deser's remarkable result predicts the nature of gravitational coupling from an invariance-breaking effect which is significantly related to the presence of background (cosmic) matter distributed in a finite universe.

In trying to understand the nature of the invariance-breaking effect, it will evidently be useful to find out how the quantum theory tries to achieve this, because the conformal transformations may well require a distinct interpretation at the quantum level. The purpose of the present paper is to undertake a consideration of this issue. We emphasize that this consideration is still far from being complete, but we hope that the paper can stimulate further investigations in this direction.

The organization of this paper is as follows: In Section 2 we present the model in terms of a conformal invariant action functional for a classical scalar field. In Section 3 we replace the classical scalar field by a quantum field and derive the general structure of quantum constraints imposed on physical states. To avoid the singular character of the latter constraints a duality principle is applied. The resulting finite structure is shown in Section 5 to admit a dimensional coupling of the states to gravity in the form of a scalar tensor theory. To arrive at this result we use restrictive conditions of direct physical meaning, which in particular provide a way to interpret the conformal transformations at the quantum level. Finally, the correspondence of quantum constraints with the Einstein equations is established by introducing an invariance-breaking effect.

## 2. THE MODEL

We shall consider a system consisting of a real scalar field  $\phi$  and the gravitational field, described by an action functional of the form

<sup>4</sup>For a general discussion of conformal invariance in the gravitational context see Beckenstein and Meisel (1980).

<sup>5</sup>Note that conformal invariance is more general than scale invariance which is used in Deser's paper. If scale invariance is characterized by vanishing of the trace of the energy-momentum tensor, conformal invariance implies scale invariance in the absence of dimensional parameters in the theory.

$$S[\phi] = -\frac{1}{2} \int d^4x \sqrt{-g} (g_{\alpha\beta} \partial^\alpha \phi \partial^\beta \phi + \frac{1}{6} R \phi^2) \tag{2}$$

where  $R$  is the scalar curvature. Note that there is no contribution of the free gravitational field to the action.

Variation with respect to  $\phi$  leads to the equation

$$(\square - \frac{1}{6} R)\phi = 0 \tag{3}$$

and that with respect to  $g_{\mu\nu}$  results in a zero constraint on the so-called conformal energy-momentum tensor, namely

$$T_{\mu\nu}[\phi] = 0 \tag{4}$$

with<sup>6</sup>

$$T_{\mu\nu}[\phi] = [\frac{2}{3} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{6} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi] - \frac{1}{3} (\phi \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \phi \nabla_\alpha \nabla^\alpha \phi) + \frac{1}{6} \phi^2 G_{\mu\nu} \tag{5}$$

Here  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$  is the Einstein tensor;  $\nabla_\mu$  indicates the covariant derivatives. One should recognize that the latter constraint is not independent of equation (2). Indeed, taking the trace of (3), one gets

$$T^\alpha_\alpha[\phi] = \phi(\square - \frac{1}{6} R)\phi = 0 \tag{6}$$

from which equation (3) can be derived. This feature is a consequence of the conformal symmetry of the action (2), which leaves us one degree of freedom unspecified. Indeed the action (2) is invariant under the conformal transformations

$$\phi \rightarrow \Omega^{-1}(x)\phi, \quad g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu} \tag{7}$$

from which it follows that various frames can be assigned to a theory defined by the action (2) depending on the particular configuration one chooses for the scalar field  $\phi$ . Different configurations can be considered as corresponding to different choices of the local standards of units. Therefore, different frames can alternatively be separated by the local values of the dimensional quantities that enter the theory.

Since the action (2) contains no absolute scale of length (a length which can be considered as constant in any conformal frame), there is nothing which incorporates a distinction between the standards of units. Therefore,

<sup>6</sup>Some authors write  $T_{\mu\nu}[\phi]$  in the alternative form

$$T_{\mu\nu}[\phi] = [\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \phi \nabla^\alpha \phi] + \frac{1}{6} (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) \phi^2 + \frac{1}{6} \phi^2 G_{\mu\nu}$$

For the application of the point splitting, see next section; this form, however, is not the convenient one, because it involves derivatives of  $\phi^2$ .

all configurations of  $\phi$  must be taken as physically equivalent. However, an important new feature arises at the quantum level if the nature of gravitational coupling is legitimately attributed to cosmological boundary conditions, in accordance with which the physical states are required to incorporate the corresponding form of large-scale correlations. Thus the possibility arises to assign a finite cosmological range to these correlations, measured by an absolute scale of length, namely the length corresponding to the physical size of the universe. As a result a preferred frame can be singled out at the quantum level of the theory.

To establish the possibility of this scenario, we shall consider  $\phi$  as a quantum field and proceed to extract the admissible form of the quantum constraints to be imposed on  $\phi$ .

### 3. QUANTUM CONSTRAINTS

If  $\phi$  is considered as a quantum field, the constraints (4) can be replaced by a set of quantum constraints to be imposed on the physically admissible states  $|\omega\rangle$  of  $\phi$ , namely

$$T_{\mu\nu}[\phi]|\omega\rangle = 0 \quad (8)$$

We shall deal with a particularly useful version of this equation in the form of constraints to be imposed on the relevant expectation values, namely

$$\langle\omega|T_{\mu\nu}[\phi]|\omega\rangle = 0 \quad (9)$$

In general, this equation corresponds to a set of singular quantum constraints, because the operator  $T_{\mu\nu}[\phi]$  involves a singular operation, namely the product of the field operator  $\phi$  at the same space-time point. The nature of the singularity is related to the short-distance singularity of the states. More specifically, we may use a symmetric splitting of the point  $\chi$  into two different neighboring points and write the constraints (9) in the form<sup>7</sup>

$$\lim_{x' \rightarrow x} D_{\mu\nu}(x, x') \langle\omega|\{\phi(x), \phi(x')\}|\omega\rangle = 0 \quad (10)$$

Here  $D_{\mu\nu}(x, x')$  is the differential operator

$$\begin{aligned} D_{\mu\nu}(x, x') &= \frac{1}{6} (g_\nu^{\nu'} \nabla_\mu \nabla_{\nu'} + g_\mu^{\mu'} \nabla_\nu \nabla_{\mu'}) - \frac{1}{12} g_{\mu\nu} g_{\alpha'}^{\alpha'} \nabla_{\alpha'} \nabla^\alpha \\ &\quad - \frac{1}{12} (\nabla_\mu \nabla_\nu + g_\mu^{\mu'} g_\nu^{\nu'} \nabla_{\mu'} \nabla_{\nu'}) \\ &\quad + \frac{1}{12} g_{\mu\nu} (\nabla_\alpha \nabla^\alpha + \nabla_{\alpha'} \nabla^{\alpha'}) + \frac{1}{12} G_{\mu\nu} \end{aligned} \quad (11)$$

<sup>7</sup>The procedure corresponds to a well-known procedure used in quantum field theory in curved space in the context of stress tensor renormalization; see, e.g., Wald (1975, 1978).

and  $g_{\mu}^{\nu}$  is the bivector of parallel transport. This expression relates the character of the operator  $T_{\mu\nu}[\phi]$  to the local structure of the symmetric two-point function.

Our objective is now to convert the constraints (10) into an admissible finite form. For this purpose we first need to introduce a duality principle in close relation to the work of Salehi (1997).

#### 4. THE PRINCIPLE OF COMMUTANT DUALITY

The formulation of this principle originates from the desire to incorporate the distinct character of large-scale (dislocalized) correlations in the gravitational context into the short-distance characteristics of the physical representations of the algebra of local observables generated by a quantized field. In conventional quantum field theory one usually assumes that correlations at arbitrary large distances have no actual influences on the short-distance characteristics of the representations. In a qualitative way it is immediately evident that this idealized setting gives rise to a corresponding conceptual shortcoming concerning the specification of the local characteristics of the representations in an infinitesimal domain. The appearance of short-distance singularities is the most dramatic aspect of such a conceptual shortcoming. If one insists on this idealized setting, a satisfactory removal of the difficulty cannot be achieved. However, to improve the predictive power of quantum field theory, kinematical criteria which control the nature of the singularities involved are usually used. We may mention, for example, the scaling criterion advanced in Haag *et al.* (1984) and Fredenhagen and Haag (19 ), which asserts that, in an infinitesimal neighborhood of a point, the local singularity of states in a physical representation should have the closest admissible correspondence to the singularity structure of the vacuum state of a free massless field in flat space. Such a criterion, important as it is for characterizing the kinematical level of the theory, is incomplete for the purpose of a dynamical incorporation of the gravitational coupling of a quantized field. In fact, it is one of the structural implication of that coupling, as furnished by the principle of general covariance, that an inescapable link must exist between the local (infinitesimal) characteristics of physical representations and large-scale correlations. The principle of commutant-duality (Salehi, 1997) makes a specific assumption to incorporate such a link. We present first a general formulation of this principle using the standard notations of the algebraic approach to quantum field theory.<sup>8</sup>

<sup>8</sup>We assume that the reader is familiar with the algebraic approach to quantum field theory (Haag, 1992).

We recall that in the algebraic approach the intrinsic mathematical description of quantum field theory is based on the correspondence between every open region  $\mathbb{O}$  in space-time and an involutive algebra  $\mathcal{A}(\mathbb{O})$ , the algebra generated by local observables which can be measured in  $\mathbb{O}$ . Given such a correspondence, the total (quasilocal) algebra of observables  $\mathcal{A}_{\text{obs}}$  can be identified with the set-theoretic union of all local algebras  $\mathcal{A}(\mathbb{O})$ , namely

$$\mathcal{A}_{\text{obs}} = \cup \mathcal{A}(\mathbb{O}) \quad (12)$$

Consider now a sequence of space-time regions  $\mathbb{O}_n$  shrinking to a point  $\chi$  as  $n \rightarrow \infty$ . For any  $\mathbb{O}_n$  consider the associated local algebra of observables  $\mathcal{A}(\mathbb{O}_n)$ . The principle of commutant duality assumes that in the limit  $n \rightarrow \infty$  one should have an exact correspondence between  $\mathcal{A}(\mathbb{O}_n)$  and the “physical” commutant of the total (quasilocal) algebra of local observables  $\mathcal{A}_{\text{obs}}$ .

The attribute “physical” means that the commutant should properly be taken as being defined with respect to an algebra larger than the total algebra of local observables. Denoting the former by  $\mathcal{A}_\infty$ , we should have the inclusion

$$\mathcal{A}_{\text{obs}} \subset \mathcal{A}_\infty \quad (13)$$

Concerning the choice of the algebra  $\mathcal{A}_\infty$ , an assumption of general nature is made. It is required (Salehi, 1997) that  $\mathcal{A}_\infty$  should essentially incorporate new dislocalized elements (elements which do not arise as images of local observation procedures) in such a way as to make the total algebra of local observables  $\mathcal{A}_{\text{obs}}$  properly correlated with dislocalized properties in space-time. This requirement can be converted into appropriate restrictions imposed on the choice of the physical states:

Let us imagine that a physical system can basically monitor all conceivable dislocalized correlations in space-time. Then it should properly be described by a state, a positive linear functional, over the large algebra  $\mathcal{A}_\infty$ . Given such a state over  $\mathcal{A}_\infty$ , we get by the GNS construction (Haag, 1962) a representation of  $\mathcal{A}_\infty$  by an operator algebra acting on a Hilbert space in which the state is represented by a cyclic vector  $\Omega$ . We shall require that the vector  $\Omega$  shall be a separating vector for the total algebra of local observables. This means that  $\Omega$  cannot be annihilated by elements of  $\mathcal{A}_{\text{obs}}$ . In this way the entire net of algebras of local observables becomes correlated with dislocalised elements of its physical commutant in the large algebra  $\mathcal{A}_\infty$ .

However, in the quantum field theory there is another level of description of a physical system, namely the conventional one in which a physical system is ideally described by a state over the total algebra of local observables  $\mathcal{A}_{\text{obs}}$ . In reality, the more information which should, in principle, be available in the form of correlations between local observables in  $\mathcal{A}_{\text{obs}}$  and the dislocalised elements of  $\mathcal{A}_\infty$  has a significant effect on the nature of the conventional description of a physical system in quantum field theory. From the principle

of commutant duality we obtain the specific treatment of this effect. In fact, using the natural inclusion properly  $\mathcal{A}(\mathbb{O}) \subset \mathcal{A}_{\text{obs}}$  for an arbitrary space-time region  $\mathbb{O}$ , it is a simple matter to infer from the commutant duality that the commutant of  $\mathcal{A}_{\text{obs}}$  with respect to  $\mathcal{A}_{\infty}$  can be reduced to a commutative algebra lying in the center of  $\mathcal{A}(\mathbb{O})$ . Thus, in essence, the principle of commutant duality asserts that a transition can be made from the physical commutant of  $\mathcal{A}_{\text{obs}}$  to a commutative algebra which lies in the center of any local algebra  $\mathcal{A}(\mathbb{O})$ . In this way, the dislocalized correlations between the entire net of local algebras  $\mathcal{A}(\mathbb{O})$  and the elements of the physical commutant of  $\mathcal{A}_{\text{obs}}$  are transferred into “classical” properties.

It should clearly be understood that this latter statement corresponds to a natural consistency requirement within the conventional description of a physical system according to which a physical system is described by a state over the algebra  $\mathcal{A}_{\text{obs}}$ . Since such states cannot monitor all conceivable dislocalized properties in the large algebra  $\mathcal{A}_{\infty}$ , we should, in fact, reject the possibility of making a distinction between different states over  $\mathcal{A}_{\text{obs}}$  by the elements of the physical commutant, a feature which obviates the need for a transition between the elements of the physical commutant to classical quantities in any space-time region. This remark shows that the adoption of the principle of commutant duality imposes a necessary restriction on the nature of the conventional description of a physical system in quantum field theory.

We should, however, emphasize the essential new ingredient which is introduced by the principle of commutant duality into the conventional description of a physical systems in quantum field theory. In fact, the distinct transition of the physical commutant of  $\mathcal{A}_{\text{obs}}$  into classical properties which is demanded by that principle implies that the strict localization of any local observable at an arbitrary point of the space-time can be reduced to a state-independent classical quantity which is generated by dislocalized properties of space-time. As a consequence, the particular value of such a quantity in a physical representation of  $\mathcal{A}_{\text{obs}}$  cannot arbitrarily be prescribed, but must depend on the actual large-scale boundary conditions imposed on that representation; it is a distinct superselection quantity that enters the dynamical description of local physics in an infinitesimal neighborhood of a point. In this way the principle of commutant duality incorporates a vital correlation between the infinitesimal characteristics of a physical representation of  $\mathcal{A}_{\text{obs}}$  and the large-scale boundary conditions characterizing that representation in the large.

## 5. FINITE STRUCTURE OF QUANTUM CONSTRAINTS

Returning to our primary objective, namely the derivation of the admissible finite structure of the constraints (10), we apply now the principle of

commutant duality to derive the gravitational coupling of the quantum field  $\phi$ . The explicit use of the commutant duality has an important consequence. In fact, if we apply the commutant duality to a representation of the algebra  $\mathcal{A}_{obs}^\phi$ , the total algebra of local observables generated by the quantum field  $\phi$ , we can reduce the operator product  $\phi(x)\phi(x')$  for a sufficiently small separation of the points  $x$  and  $x'$  to a classical quantity which is generated by the particular form of the large-scale boundary conditions imposed on that representation. Thus, it follows from the application of the commutant duality that, given a representation of  $\mathcal{A}_{obs}^\phi$ , the dimensional quantity resulting from the coincidence limit

$$\lim_{x' \rightarrow x} \langle \omega | \phi(x)\phi(x') | \omega \rangle \quad (14)$$

can be as taken as the parameter of the dimensional coupling of states to the large-scale boundary conditions defining that representation in the large. However, to accurately attribute this coupling to the gravitational coupling, we must first look for a natural rule connecting a change of the physical situation in the large to the local action of the conformal transformations. The need for such a rule arises from the observation that the parameter of the dimensional coupling of states over  $\mathcal{A}_{obs}^\phi$  to the large-scale boundary conditions, which is taken as embodied in the coincidence limit (14), will in general change if a transition is made from a unitary class of representations of  $\mathcal{A}_{obs}^\phi$  to another one with a different set of large-scale boundary conditions. On the other hand, since a dimensional coupling parameter will also be affected by the action of the conformal transformations, such a transition between inequivalent representations should be expected to be linked with a corresponding conformal transformation affecting the coincidence limit (14). Thus we may legitimately look for a natural rule connecting a change of the large-scale boundary conditions to the local action of conformal transformations.

In accordance with this observation, different (unitary inequivalent) representations of  $\mathcal{A}_{obs}^\phi$  should be related by the local action of conformal transformations on the quantity defined by the coincidence limit (14). The accurate formulation of the rule is the following: Given a representation of  $\mathcal{A}_{obs}^\phi$ , we define, in the first place, a c-number field  $\psi$ , the universal one-point function of the quantum field  $\phi$  in that representation, in terms of the coincidence limit (14) by the condition

$$\lim_{x' \rightarrow x} \langle \omega | \phi(x)\phi(x') | \omega \rangle = \psi^2(x) \quad (15)$$

In the second place, we relate the arbitrariness in choosing the representation of  $\mathcal{A}_{obs}^\phi$  from which one starts to the freedom to change the configuration of



the universal one-point function  $\psi$  and the metric tensor  $g_{\mu\nu}$  according to the law

$$\psi \rightarrow \Omega^{-1}(x)\psi, \quad g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu} \tag{16}$$

which is the analogue of (7). This is a statement about the actual description of the conformal transformations in the model under consideration.

Let us now define the (two-point) correlation function

$$\langle \omega | \phi(x)\phi(x') | \omega \rangle_c = \langle \omega | \phi(x)\phi(x') | \omega \rangle - \psi(x)\psi(x') \tag{17}$$

In terms of  $\psi$  and  $\langle \omega | \phi(x)\phi(x') | \omega \rangle_c$  the quantum constraints (10) can be written as

$$G_{\mu\nu} = -6\psi^{-2} [Q_{\mu\nu}\{\psi\} + S_{\mu\nu}\{\omega\}] \tag{18}$$

where

$$Q_{\mu\nu}\{\psi\} = \left[ \frac{2}{3} \nabla_\mu \psi \nabla_\nu \psi - \frac{1}{6} g_{\mu\nu} \nabla_\alpha \psi \nabla^\alpha \psi \right] - \frac{1}{3} (\psi \nabla_\mu \nabla_\nu \psi - g_{\mu\nu} \psi \nabla_\alpha \nabla^\alpha \psi) \tag{19}$$

and

$$S_{\mu\nu}\{\omega\} = \lim_{x' \rightarrow x} \tilde{D}_{\mu\nu}(x, x') \langle \omega | \{ \phi(x)\phi(x') \} | \omega \rangle_c \tag{20}$$

In the last equation  $\tilde{D}_{\mu\nu}(x, x')$  is given by the differential operator (11) without the  $G_{\mu\nu}$  term. These equations describe the coupling of the states in a representation with the universal boundary conditions defining that representation. We infer that the latter coupling takes the form of a gravitational coupling in a scalar tensor theory, the tensor  $S_{\mu\nu}\{\omega\}$  describing the state-dependent distribution of matter. One should recognize, however, that the nature of the tensor  $S_{\mu\nu}\{\omega\}$  is still unspecified. Our objective is now to derive a general restriction to be imposed on that tensor from an invariance-breaking argument.

## 6. INVARIANCE-BREAKING EFFECT

Using the principle of commutant duality, it was shown that for the two-point function the short-distance limit (14) in a physical representation of  $\mathcal{A}_{obs}^\phi$  can be characterized in terms of a universal one-point function  $\psi$ , the latter being generated by the dynamical influence of the large-scale boundary conditions imposed on that representation. As a consistency requirement, we have proposed a correspondence between a conformal transformation affecting the universal one-point function  $\psi$  and a transition between inequivalent global representations of the algebra  $\mathcal{A}_{obs}^\phi$ .

If no distinction could be made between different inequivalent global representations of  $\mathcal{A}_{obs}^\phi$  from the physical point of view, all conformally related configurations of  $\psi$  should be considered as equivalent. However, such a distinction can actually be made by the boundary conditions of the universe:

Consider, namely, that class of representations of  $\mathcal{A}_{obs}^\phi$  for which the field  $\psi$  is assumed not to be generated by the global boundary conditions in nearby regions<sup>9</sup> of space-time, but only by the asymptotic (cosmological) tail of boundary conditions at very distant regions of space-time.<sup>10</sup> This class of representations, which is called in the following the absolute class, is distinctly preferred, because the corresponding configuration of  $\psi$  is sensitive only to cosmological properties of space-time which are fixed in a universal manner. In particular, those conformal transformations which are connected to a change of the global boundary conditions in nearby regions of space-time will not affect the function  $\psi$ . This function must therefore be in its absolute (constant in any conformal frame) configuration. The existence of such an absolute configuration of  $\psi$  may be considered as an invariance-breaking effect.

Our objective now is to estimate the value of  $\psi$  in its absolute configuration. First we legitimately require that the dynamical excitation of  $\psi$  in the absolute configuration should be connected to larger-scale characteristics of the universe. To accurately use this requirement, let us take the trace of the equations (18) to obtain

$$(\square - \frac{1}{6}R + \psi^{-2}S_\alpha^\alpha\{\omega\})\psi = 0 \quad (21)$$

This equation shows that the term  $\psi^{-2}S_\alpha^\alpha\{\omega\}$  acts as a (dynamical) mass term which characterizes the massive excitations of  $\psi$ . Since the dynamical excitations of  $\psi$  in the absolute configuration should be connected to large-scale characteristics of the universe, the mass term  $\psi^{-2}S_\alpha^\alpha\{\omega\}$  should be measured by the cosmological constant  $\Lambda$ . This results in a relation of the type

$$\psi^{-2} \sim \Lambda/S_\alpha^\alpha\{\omega\} \quad (22)$$

Correspondingly, we should measure  $S_\alpha^\alpha\{\omega\}$  by the energy density of the larger-scale distribution of matter. Now approximating the cosmological constant by its observational bound  $\Lambda_{obs} \sim L^{-2}$ , where  $L$  is the size of the universe ( $L \sim 10^{29}$  cm), and taking into account the remarkable empirical fact that the energy density of the large-scale distribution of matter in the universe coincides (in a rough order-of-magnitude manner only) with the

<sup>9</sup>“Nearby regions” means distant regions without extension to cosmological distances.

<sup>10</sup>This limitation gives an expression to Mach’s principle. For a classical exposition of this principle see Brans and Dicke (1961).

contribution of  $\Lambda_{obs}$  to the vacuum energy density, we get in the absolute configuration  $\psi$  the remarkable correspondence between  $\psi^2$  and the inverse value of the gravitational constant  $(8\pi G)^{-1}$ . As a consequence, equations (18) reduce to a set of the Einstein equations.

## 7. CONCLUDING REMARKS

We have seen that the absolute configuration of  $\psi$  leads to a reasonable prediction of the dynamical coupling of states to gravity. This was possible because the latter configuration was established in the absolute class of representations of the algebra  $\mathcal{A}_{obs}^\phi$ , in which  $\psi$  depends only on the asymptotic tail of boundary conditions at distant regions of space-time.

It should be realized that there are a variety of unitarily inequivalent representations in the absolute class of representations of  $\mathcal{A}_{obs}^\phi$ . In the present theory different representations in the absolute class will differ with respect to global boundary conditions in nearby regions of space-time. But they all coincide in the configuration of  $\psi$ .

In general if a transition is made between different representations in the absolute class, the physical content of the theory may change. This effect may be of particular importance in studying the still-unsolved problem of backreaction (Salehi, 1992) that is the response of the metric to a particular configuration of nearby boundary conditions. The exploration of this aspect is an interesting subject which requires considerable clarification. We hope to address the issue in a future publication.

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